Grokking and Logical Systems

by Fernando - Friday, March 27, 2015

http://www.ghyzmo.com/grokking-and-logical-systems/

Those who cannot learn from history are doomed to repeat it.

—George Santayana(Wikiquote)

Mathematics went through a period of development, the history of which demonstrates the difference between grokking and knowing by logical connectivity. Now the curriculum of common core mathematics repeats the mistakes of the past in mathematics, which is bound to generate confusion rather than understanding among students. That there are problems involved in modern methods of education is demonstrated by a this Ted talk by Carol Dweck.

Background

I was always intrigued by Kurt Gödel and his incompleteness theorem that put an end to all kinds of mathematical shenanigans. Mathematicians were trying to reconstruct mathematics on a strictly logical and self consistent basis starting from a few definitions and postulates. The idea was to follow Euclid' s lead in his systematization of Egyptian geometry, which is called Euclidean Geometry. Euclid' s great work was a beautiful piece of mathematical architecture admired by most practitioners and academics. So geometry became the model for developing a beautiful and elegant branch of mathematics.

Anyway, what mathematicians were trying to do is to develop all of mathematics the same way as Euclidean geometry, beginning from the definition of of numbers themselves, building up the rules of arithmetic, and finally graduating to the big stuff such as calculus, Hilbert spaces and Grassmann algebras . At one time I was an avid reader of the works of **Henri Poincaré** who severely criticized the works of the logicians trying to reinvent number theory a la Euclid. I can remember his criticisms because they were very humorous.

The description of **Poincaré** 's criticisms of Euclidophilic tilting at arithmetic windmills are described in a more staid way by the hyperlinked article in Wikipedia (under his name above), as follows:

> Poincaré believed that arithmetic is a synthetic science. He argued that Peano's axioms cannot be proven non-circularly with the principle of induction (Murzi, 1998), therefore concluding that arithmetic is a priorisynthetic and not analytic. Poincaré then went on to say that mathematics cannot be deduced from logic since it is not analytic. His views were similar to those of Immanuel Kant (Kolak, 2001, Folina 1992). He strongly opposed Cantorian set theory, objecting to its use of impredicative definitions.

What Kurt Gödel Did to Tip over the Axiomatic Cart

Kurt Gödel, a good friend and walking companion of of Albert Einstein (see Palle Yourgrau' "A World Without Time: The Forgotten Legacy of Godel and Einstein") was an expert and adept logician. By employing the same set of tools used by those who hoped to reduce the basis of mathematics to logic, Gödel proved that their efforts were futile. He proved that no closed logical system can be built up from a finite set of well defined axioms and definitions, because it cannot be both self consistent and complete. The truth of such a system requires it to be open to a least one more axiom, which is a recursive way of saying that an infinite set of axioms are required. The following quote is from a Wikipedia article hyperlinked to Kurt Gödel :

> **Gödel's incompleteness theorems** are two theorems of mathematical logic that establish inherent limitations of all but the most trivial axiomatic systems capable of doing arithmetic. The theorems, proven by Kurt Gödel in 1931, are important both in mathematical logic and in the philosophy of mathematics. The two results are widely, but not universally, interpreted as showing that Hilbert's program to find a complete and consistent set of axioms for all mathematics is impossible, giving a negative answer to Hilbert's second problem.

Relation to Grokking

Having been a reader of Science Fiction since I was a child, I used an old term, "grok",coined by Robert A. Heinlein. He used it to describe a process of learning by deep and intimate understanding, which is a process that I have used in my life to boot strap my acquisition of knowledge and skills. Here are a definition of the term from a dictionary online,

> /grok/, /grohk/ (From the novel "Stranger in a Strange Land", by Robert A. Heinlein, where it is a Martian word meaning literally "to drink" and metaphorically "to be one with")
> 1. To understand, usually in a global sense. Connotes intimate and exhaustive knowledge. *[Here emphasized in red by me.]*
> Contrast zen, which is similar supernal understanding experienced as a single brief flash. See also glark.
> 2. Used of programs, may connote merely sufficient understanding. "Almost all C compilers grok the "void" type these days."
> [Jargon File]
> (1995-01-31)

The way I have used "grok [See for Webster's definition]" is as a verb in the sense described as number one above, which is emphasized in red. The second definition is more one used by programmers and is a bit of jargon, in that sense. The term (n. 1 definition) is very handy in simplifying and shortening discussions, such as in the issue of the axiomatic remodeling of arithmetic. For example, we could say that Poincaré thought that arithmetic defied logic, and needed to be grokked instead.

An Axiomatic System is a Mesh of Points

By thinking abstractly, we can grok something of the implication of Gödel's proofs. Picture the space of truth of a mathematical system as a portion of some kind of continuous surface. A surface consists of an infinity of points. A logical system tries to reduce the truth of that area of mathematics to a discrete and finite set of propositions and definitions. Thus anyone trying to describe the surface of mathematical truth by such a mesh of points, ends in the futile effort of trying to describe a continuous space with a finite mesh of points. This will be a never ending effort, stretching out to infinity. The only way of getting into the system is through some kind of intimate and exhaustive knowledge, sufficient to see the whole surface in the mind's eye--that is, **to grok** it.

Sharing Mathematical insights with Extraterrestrials

The continuous surface of truth coverage by a network of a finite number of points of logic indicates a problem in communicating our understanding to an extraterrestrial civilization (extraterrestrials, 'ET' s), should we ever establish communications with one. The alien humanoids may not understand Euclidean geometry in terms of the same system of axioms and definitions that we used, but which cover the same surface of understanding through an alternate network of logic. Or perhaps, they have moved beyond axiomatic systems in which they can see the surface itself i.e., they grok it.

Conclusion

In high school my favorite subjects were the sciences and mathematics. Euclidean geometry was fun. I really dug it. Each proof was a challenge, like a crossword puzzle to be worked out logically.

The geometry teacher tried to explain Euclid very thoroughly, dwelling over each proof an interminable length of time. She was a good teacher, but very rule-bound. I would sit in the back of the class and doodle, because I did not want t0 ruin the fun of my working out the proof. She would become exasperated with me and tried to catch me off guard. In the middle of long proof, she would suddenly say, "Fernando, You come up to the board and finish this proof." So I would go up to the board and not just finish , but starting over again, I would do the proof in fewer steps, than what the teacher had already done but not finished. Other times, I would weave a proof that the was for the teacher a short proof, in many more steps, as I explored on my feet at the board the intricacies of the network of Euclidean logic.

During high school, I may have encountered the term, "grokking", but it did not register in my conscious thoughts. But, that was exactly what I was doing with Euclidean geometry. Somehow, I could see in my mind's eye the landscape covered by the system of logic, and used a series of logical jumps connecting the points of the mesh to move around over this surface. The proof was just a record of this grokking journey.

Grokking Physics

Unknowingly, I had taken the path of grokking in high school because I had a spastic memory. I could remember bits and pieces, but could not recall what I needed on demand. So I had sort of stumbled upon what I can now name as "grokking" as a means of overcoming a spastic memory. In the case of the value of grokking, the proof was in the pudding. Once I grokked an area of physics, it became a spring of knowing for me. Owing to grokking, when I took physics in high school, I aced all the examinations,

getting straight 'A^+'s (the highest scores) consistently.

When I was in graduate school, I grokked advanced physics, and from that basis. I derived the necessary equations and approximations to solve a variety of problems in physics based on grokking. I looked at many of my fellow students, feeling sorry for their efforts to get by in physics by memorizing derivations, and remembering how to solve problems.

The night before my PhD qualifying examination, I saw my advisor and waved at him across a crowded room of people at intermission at a local theater. I had a date with me, a tall blonde girl. The next day I began taking the examination, which continued the another day. During that period, several of the graduate students stayed in the exam room to review their notes and brush up on the next topic. I made sure that I went out for a good lunch, and relaxed in the evenings at leisure. The upshot of this approach was that I passed the examination with honors.

Why is Grokking important in Modern Times?

During my period of working for someone else, the idea of **information overload** was discussed widely. Look at! It is just streams of information comming at you from from every direction you turn on the Internet. To worsen the prospect, is that a lot of misinformation is also circulated along with real information. In that case, the more people look into a subject, particularly if it is controversial, the more they are likely to get confused. But, there is a solution out of the information overload mess, for those who can get a hang of the process of grokking. Which brings up the dismal state of public education.

A little knowledge is a dangerous thing.

One thing that comes to mind is the old warning "A little knowledge is a dangerous thing.", which applies to modern eduction of the innocent, such as in the "common core curriculum" for arithmetic. It seems to me that "educators" in this area are trying to reduce arithmetic to a set of axioms and definitions." They are trying to force students through a laborinth of unnecessary thinking just to get through a simple arithmetic problem. The result of these efforts is the flourishing of private schools, and therefore, an increasing gap between haves and have-nots, a problem which inept politicians try to solve by throwing money at the problem.

Doing arithmetic is like piano students doing their scales at the piano. Through the repetition of traversing the notes of the musical scales, they develop a familiarity with the lay of the musical land that allows them to grow into good musicians. In this way they grok the piano. A wrong way to learn to do music is by reading a lot of music theory, and then being told that the rest of learning to play the piano is just an unfolding the musical logic.

HAPPY GROKKING.

PDF generated by Kalin's PDF Creation Station